Interval Prediction for Exponential Smoothing

Abstract

The exponential smoothing emerged in the second half of the 20th century as a time series prediction technique, since then, much has been developed in the area and the method has been extended to include series which contain trend and seasonality. Besides that, methods of calculating prediction intervals have also been developed. A brief revision of the parametric prediction intervals for linear exponential smoothing methods is made. New non-parametric prediction intervals are developed, using the bootstrap technique. Tests are performed with M3 competition data and with simulated series.

Methods

The simple exponential smoothing uses a weighted average to produce a forecast. The weights decrease exponentially, hence the name exponential smoothing. There are also methods of exponential smoothing that include trend and seasonality:

Seasonality	Trend: N	Trend: A	Trend: A_d
N	$Z_t = l_{t-1} + a_t$ $l_t = l_{t-1} + \alpha a_t$	$Z_t = l_{t-1} + b_{t-1} + a_t$ $l_t = l_{t-1} + b_{t-1} + \alpha a_t$ $b_t = b_{t-1} + \alpha\beta a_t$	$Z_t = l_{t-1} + \phi b_{t-1} + a_t$ $l_t = l_{t-1} + \phi b_{t-1} + \alpha a_t$ $b_t = \phi b_{t-1} + \alpha\beta a_t$
A	$Z_t = l_{t-1} + s_{t-m} + a_t$ $l_t = l_{t-1} + \alpha a_t$ $s_t = s_{t-m} + \gamma a_t$	$Z_t = l_{t-1} + b_{t-1} + s_{t-m} + a_t$ $l_t = l_{t-1} + b_{t-1} + \alpha a_t$ $b_t = b_{t-1} + \alpha\beta a_t$ $s_t = s_{t-m} + \gamma a_t$	$Z_t = l_{t-1} + \phi b_{t-1} + s_{t-m} + a_t$ $l_t = l_{t-1} + \phi b_{t-1} + \alpha a_t$ $b_t = \phi b_{t-1} + \alpha\beta a_t$ $s_t = s_{t-m} + \gamma a_t$

Prediction intervals

There are several ways to produce prediction intervals for these methods, such as:

- Parametric interval;
- Plugin interval;
- Bootstrap interval;

Parametric interval

We can use the state-space framework to generate parametric prediction intervals based on the normal distribution:

$$IP(Z_{t+k}, \alpha) = \hat{z}_{t_k} \pm q_{\frac{1-\alpha}{2}} \sqrt{\sigma_a^2}$$

We could also use the t distribution, but chose not to do so.

Plugin interval

To generate a prediction interval with confidence level α for k steps ahead, we simply use the quantiles from the errors calculated k steps ahead.

$$IP(Z_{t+k}, \alpha) = \left[\hat{z}_{t_k} + \hat{F}_a^{-1}\left(\frac{1-\alpha}{2}\right), \hat{z}_{t_k} + \hat{F}_a^{-1}\left(1 - \frac{1-\alpha}{2}\right) \right]$$

Bootstrapping algorithm

We use the following algorithm to calculate bootstrap samples:

1. Estimate the parameters and calculate the residuals k steps ahead;
2. Sample from the residuals and construct a new series;
3. Estimate again to obtain bootstrap parameters and a new set of residuals;

We use the quantiles from the new sets of the residuals obtained in step 3.

Tests

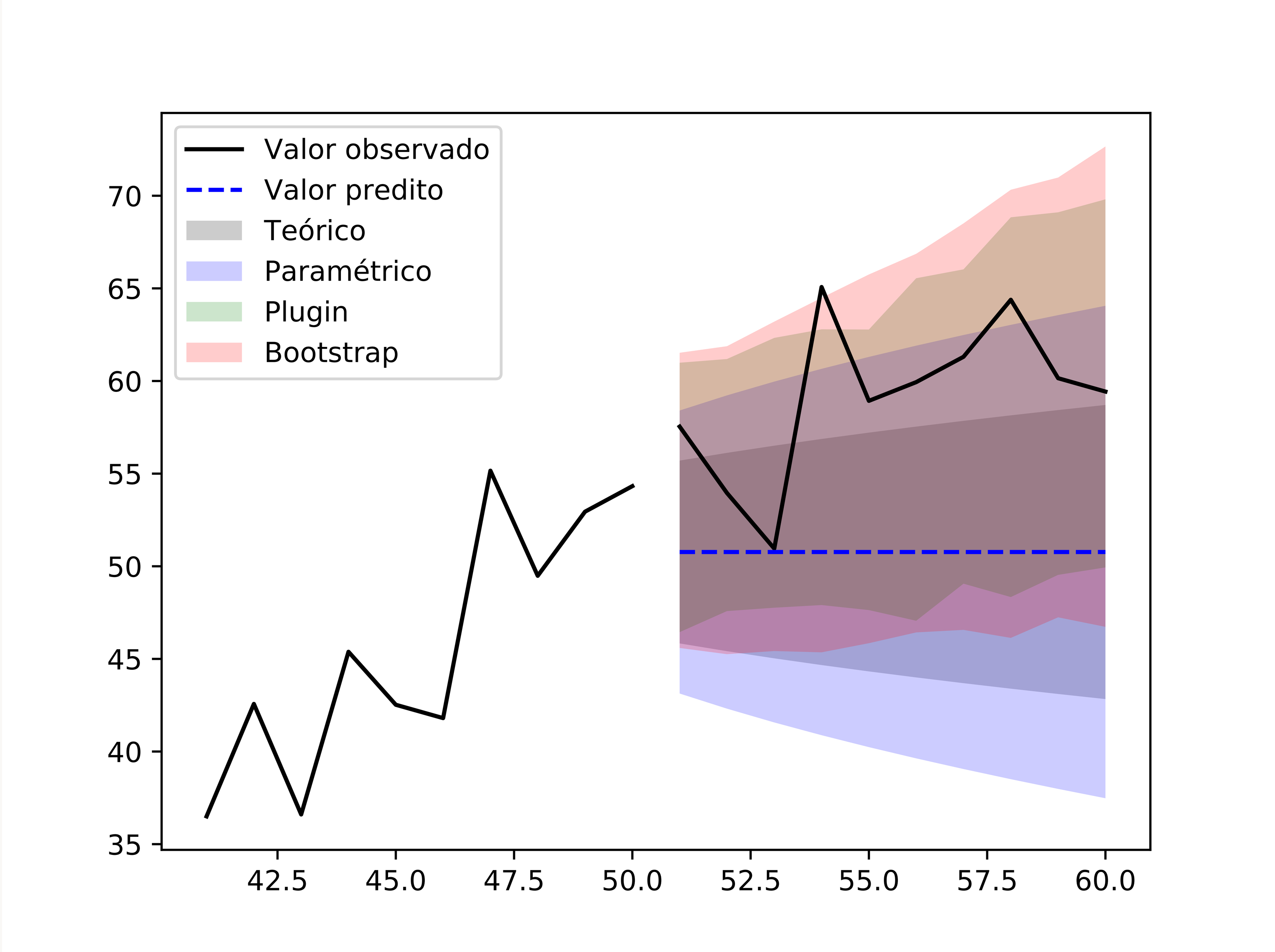
A time series from each method was simulated, with the following parameters:

- $n = 50$;
- $Var(a_t) = 9$;
- $k = 10$;
- $m = 7$ for the seasonal methods;

For testing purposes, we used 100 bootstrap replications.

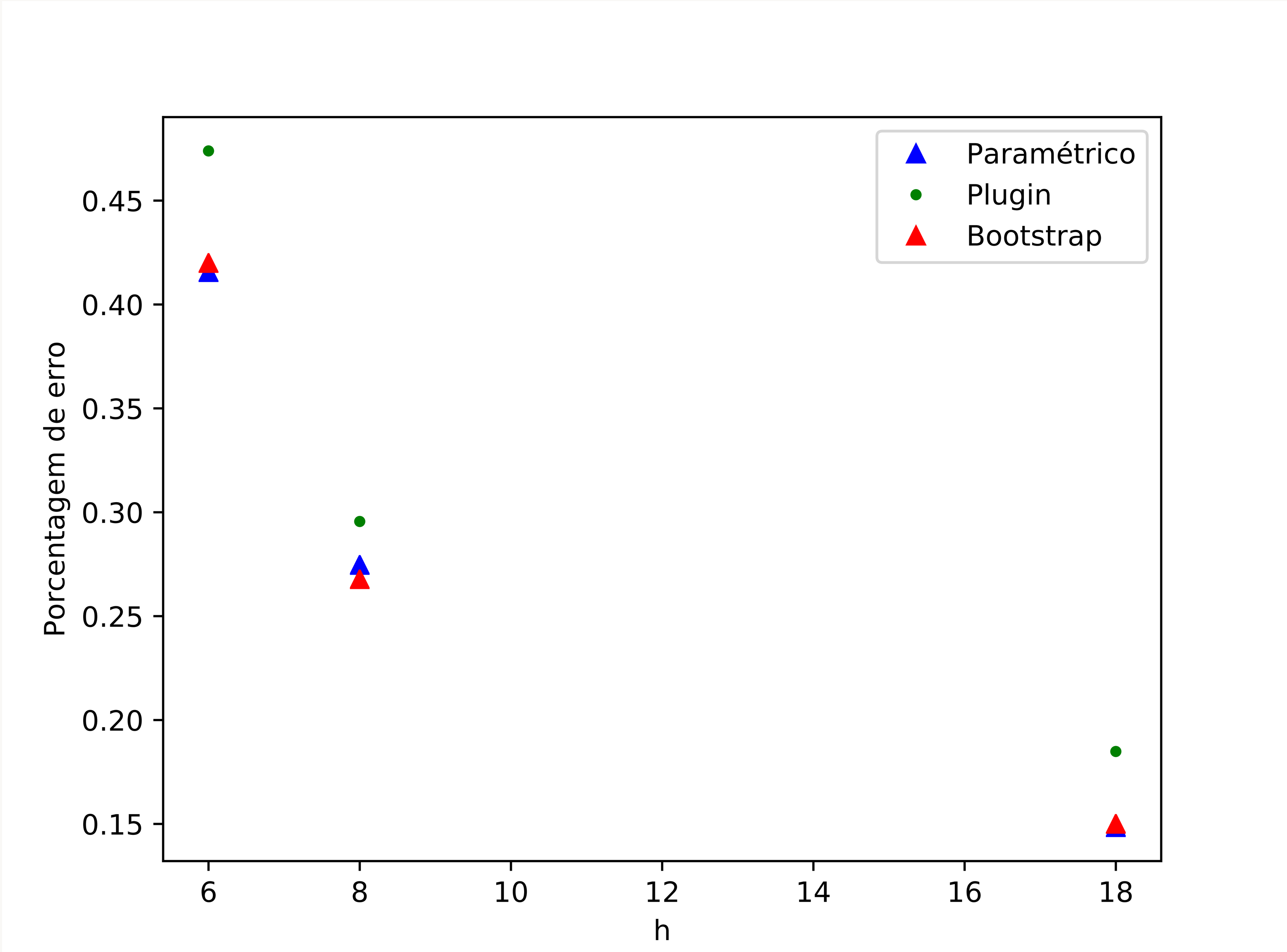
Simulated data

For the simple exponential smoothing, we obtained the following intervals:



M3 competition data

This is the comparison of the miss ratios of the three methods by forecast horizon:



Conclusions

- There are no prediction intervals that outperform the others;
- Plugin intervals can and should be used as benchmark regarding the interval amplitudes;
- The coverage probabilities of the intervals always seems to be less than the confidence;

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